NDU

MAT 224

Calculus IV

Exam # 2

Tuesday January 13, 2015

Duration: 65 minutes

Name: ______________________

Section: ____________________

Grade: ____________________

Problem Number	Points	Score
1	36	
2	17	
3	11	
4	16	
5	30	
Total	110	

1) **(36 points)** For each of the following multiple-choice questions, circle the **letter** of the correct answer. If more than one letter is circled in the same problem, you will receive no credit for that problem.

Question A **(8 points)** Using the method of Lagrange Multipliers to find the point closest to the origin on the curve of intersection of the plane $x+y+z=1$ and the cone $z^2=2x^2+2y^2$, can yield the system of equations:

a)
$$2x=\lambda+4\mu x$$
$$2y=\lambda+4\mu y$$
$$2z=\lambda$$
$$x+y+z-1=0$$
$$2x^2+2y^2-z^2=0$$

b)
$$2x=4\mu x$$
$$2y=4\mu y$$
$$2z=\lambda$$
$$x+y+z-1=0$$
$$2x^2+2y^2-z^2=0$$

c)
$$2x=\lambda+4\mu x$$
$$2y=\lambda+4\mu y$$
$$2z=\lambda-2\mu z$$
$$x+y+z=0$$
$$2x^2+2y^2=0$$

d)
$$2x=\lambda+4\mu x$$
$$2y=\lambda+4\mu y$$
$$2z=\lambda-2\mu z$$
$$x+y+z-1=0$$
$$2x^2+2y^2-z^2=0$$

Question B **(7 points)**

$$\int_0^9\int_{\sqrt{y}}^3 3\sec^2(x^3)\,dx\,dy=$$

a) $\tan(27)$
b) $\tan(8)$
c) $\tan(1)$
d) 0

Question C **(7 points)**

Let R be the region in the first quadrant of the xy-plane that lies outside the circle $x^2+y^2=1$ and inside the circle $x^2+y^2=9$. Then $\iint_R (x+y)\,dx\,dy=$

a) $\frac{2}{3}$
b) $\frac{52}{3}$
c) $\frac{54}{3}$
d) 0

Question D **(14 points)** Consider the integral $\iint_R y^3\sqrt{x-y}\,dydx$, where R is the triangular region in the xy-plane bounded by the lines $y=0, y=x$, and $x+2y=9$. Let G be the region in the uv-plane which is the image of R under the transformation $x=u+9v$ and $y=u$. Then

Part 1 $\iint_R y^3\sqrt{x-y}\,dydx =$

a) $\iint_G 27u^3\sqrt{v}\,dudv$

b) $\iint_G -27u^3\sqrt{v}\,dudv$

c) $\iint_G 3u^3\sqrt{v}\,dudv$

d) $\iint_G -3u^3\sqrt{v}\,dudv$

Part 2 The region G in the uv-plane is bounded by the lines:

a) $u=0, u=v, u+2v=9$

b) $u=0, u=v, v=3$

c) $u=0, v=0, u+3v=3$

d) $u=0, v=0, u+2v=9$

2) (17 points) Consider the region D in space that is bounded from below by the xy-plane, from above by the paraboloid $z = 2 - x^2 - y^2$ and laterally by the cylinder $x^2 + y^2 = 1$.

a) (3 points) Draw the region D.

b) (6 points) Set up triple integral for the volume of D in cylindrical coordinates according to the order of integration $dz dr d\theta$ **.**

c) (8 points) Set up triple integral for the volume of D in cylindrical coordinates according to the order of integration $dr dz d\theta$ **.**

2) (11 points) Let R be the region in the first quadrant bounded by the curves $y = 0$, $(x-2)^2 + y^2 = 4$ and $x = 2$.

a) (3 points) Draw the region R in the xy-plane.

b) **(8 points)** Set up a double integral in polar coordinates using the order of integration $drd\theta$ equal to $\iint_R f(x,y)\,dA$, where $f(x,y) = x^4 + y^3$.

4) (16 points) Let D represent the region in the first octant bounded by the coordinate planes and the planes $3x + z = 6$ and $y + z = 6$. Set up triple integrals in rectangular coordinates representing the volume of D according to each of the following orders:

a) (6 points) $dy\,dx\,dz$

b) (10 points) $dz\,dy\,dx$

5) (30 points) Let D be the solid region in space above the plane $z = 2$ and under the sphere $x^2 + y^2 + z^2 = 16$.

a) (3 points) Draw the region D.

b) (8 points) Set up triple integrals in spherical coordinates for the volume of D using the order of integration $d\rho d\phi d\theta$.

c) (12 points)) Set up triple integrals in spherical coordinates for the volume of D using the order of integration $d\phi d\rho d\theta$.

d) (7 points) Use part (b) to find the volume of D.